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TOP PAIR PRODUCTION IN e^+e^- AND $\gamma\gamma$ PROCESSES

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We analyze spin correlations between top quark and anti-topquark produced at polarized e^+e^- and $\gamma\gamma$ colliders. We consider a generic spin basis to find a strong spin correlation. Optimal spin decompositions for top quark pair are presented for e^+e^- and $\gamma\gamma$ colliders. We show the cross-section in these bases and discuss the characteristics of results.

1 Introduction

In 1994, the top quark was discovered by the CDF and D0 collaborations^{1,2}. The measured top quark mass is approximately 175 GeV, which is nearly twice the mass scale of electro-weak symmetry breaking. So the top quark with large mass brings a good opportunity to understand electro-weak symmetry breaking and to search hints of any new physics. The top quark decays electroweakly before hadronizing because its width is much greater than the hadronization timescale set by Λ_{QCD} ³. Therefore, there are significant angular correlations between the decay products of the top quark and the spin of the top quark. These angular correlations depend sensitively on the top quark couplings to the Z boson and photon, and to the W boson and b quark. This aspect is unique for the top quark sector. Actually, there are many works⁶ on the angular correlations for the top quark events produced at e^+e^- colliders. In most of these works, the top quark spin is decomposed in the helicity basis. However, G. Mahlon and S. Parke⁴ suggested a more optimal decomposition of the top quark spin to find spin correlations at hadron colliders. S. Parke and Y. Shadmi⁵ extended their work to the e^+e^- annihilation process at the leading order in perturbation theory and concluded that the “Off-Diagonal” basis is the best decomposition of top pair spins.

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In this paper we focus on the issue of what is the optimal decomposition of the top quark spins produced at e^+e^- and $\gamma\gamma$ colliders^{7,8}. At e^+e^- colliders, we analyze the differential cross-sections for the top quark pair production including QCD one-loop corrections in the soft gluon approximation. We give the differential cross-sections in a generic spin basis. The optimal spin decomposition for the top quark pair and the differential cross-section in this basis are presented. At $\gamma\gamma$ colliders, we discuss the top pair production with the circular polarized photon beams which correspond to the total angular momentum $J_z = 0, 2$. We develop analogous analyses to the case of the e^+e^- colliders and propose an useful spin basis to investigate the spin correlations.

2 Spin Correlations at e^+e^- Colliders

We present the cross-section for the polarized top quark pair production including the QCD one-loop correction in the soft gluon approximation. We show the differential cross-section for a generic spin basis. The generic spin basis here is based on the following features: Firstly, we do not discuss the transverse polarization of the top quark because it may be much smaller than the longitudinal polarization. Secondly CP is conserved. Therefore, we have defined our generic spin basis so that the spins of the top and anti-top quark are in the production plane. Then, we introduce only one parameter ξ to define the spins of top and anti-top quarks. The definition of ξ is as follows [Figure 1]: We decompose the top quark spin along the direction \vec{s}_t in the rest frame of the top quark, where it has a relative angle ξ clock-wisely to the anti-top quark momentum. Similarly the anti-top spin is defined in the anti-top quark rest frame along the direction $\vec{s}_{\bar{t}}$, which makes the same angle ξ with the top quark momentum.

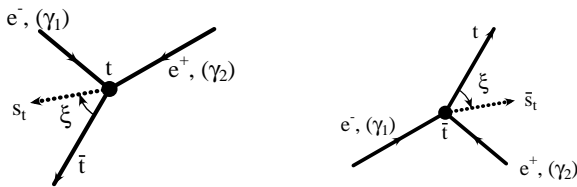


Figure 1: The generic spin for the top (anti-top) quark in the top (anti-top) quark rest frame. \vec{s}_t ($\vec{s}_{\bar{t}}$) represents the spin axis of the top (anti-top) quark.

The cross-sections in the Center of Mass (CM) frame are the functions of the scattering angle θ , the top and anti-top quark speed β and the angle ξ which defines the spin axis. In the soft gluon approximation, the differential

cross-sections at the QCD one-loop level are given by

$$\begin{aligned}
& \frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \rightarrow t_\uparrow \bar{t}_\uparrow \text{ and } t_\downarrow \bar{t}_\downarrow) \\
&= \left(\frac{3\pi\alpha\beta}{2s} \right) (A_{LR} \cos \xi - B_{LR} \sin \xi) \\
&\quad \times [(A_{LR} \cos \xi - B_{LR} \sin \xi)(1 + \hat{\alpha}_s S_I) \\
&\quad \quad - 2(\gamma^2 A_{LR} \cos \xi - \bar{B}_{LR} \sin \xi) \hat{\alpha}_s S_{II}] \quad (1)
\end{aligned}$$

$$\begin{aligned}
& \frac{d\sigma}{d\cos\theta}(e_L^- e_R^+ \rightarrow t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow) \\
&= \left(\frac{3\pi\alpha\beta}{2s} \right) (A_{LR} \sin \xi + B_{LR} \cos \xi \pm D_{LR}) \\
&\quad \times [(A_{LR} \sin \xi + B_{LR} \cos \xi \pm D_{LR})(1 + \hat{\alpha}_s S_I) \\
&\quad \quad - 2(\gamma^2 A_{LR} \sin \xi + \bar{B}_{LR} \cos \xi \pm \bar{D}_{LR}) \hat{\alpha}_s S_{II}] \quad (2)
\end{aligned}$$

The state t_\uparrow/t_\downarrow refers to the top quark with the spin in the direction $+\vec{s}_t/-\vec{s}_t$ respectively. Here α is the QED fine structure constant, $\hat{\alpha}_s = (C_2(R)/4\pi)\alpha_s$, $C_2(R) = (N_c^2 - 1)/(2N_c)$, N_c is the number of color. The kinematical variables are $\beta = \sqrt{1 - 4m^2/s}$, $\gamma = 1/\sqrt{1 - \beta^2}$. The quantity A , B , D , \bar{B} and \bar{D} are as follows;

$$\begin{aligned}
A_{LR} &= [(f_{LL} + f_{LR})\sqrt{1 - \beta^2} \sin \theta]/2, \\
B_{LR} &= [f_{LL}(\cos \theta + \beta) + f_{LR}(\cos \theta - \beta)]/2, \\
D_{LR} &= [f_{LL}(1 + \beta \cos \theta) + f_{LR}(1 - \beta \cos \theta)]/2, \\
\bar{B}_{LR} &= [f_{LL}(\cos \theta - \beta) + f_{LR}(\cos \theta + \beta)]/2, \\
\bar{D}_{LR} &= [f_{LL}(1 - \beta \cos \theta) + f_{LR}(1 + \beta \cos \theta)]/2,
\end{aligned} \quad (3)$$

with

$$f_{IJ} = Q_\gamma(e)Q_\gamma(t) + Q_Z^I(e)Q_Z^J(t) \frac{1}{\sin^2 \theta_W} \frac{s}{s - M_Z^2}.$$

Where \sqrt{s} is the CM energy, M_Z is the mass of Z boson, the angle θ_W is the Weinberg angle, and the suffix $I, J \in (L, R)$. The electron (top quark) coupling to the photon is $Q_\gamma(e)$ ($Q_\gamma(t)$). The left-handed electron (top quark) coupling to the Z boson is given by $Q_Z^L(e)$ ($Q_Z^L(t)$) and the right-handed electron (top quark) coupling to the Z-boson is given by $Q_Z^R(e)$ ($Q_Z^R(t)$). In the above calculation, we have neglected the Z boson width since the production threshold for the top quarks is far above the Z boson mass. The QCD corrections S_I

and S_{II} read

$$S_I = 2\left(\frac{1+\beta^2}{\beta} \ln \frac{1+\beta}{1-\beta} - 2\right) \ln \frac{4\omega_{max}^2}{m^2} - 8 + 2\frac{3+2\beta^2}{\beta} \ln \frac{1+\beta}{1-\beta} \\ + 2\frac{1+\beta^2}{\beta} [2\zeta(2) + 4\text{Li}_2 \frac{1-\beta}{1+\beta} + \ln \frac{1-\beta}{1+\beta} (3 \ln \frac{2\beta}{1+\beta} + \ln \frac{2\beta}{1-\beta})], \quad (4)$$

$$S_{II} = \frac{1-\beta^2}{\beta} \ln \frac{1+\beta}{1-\beta}, \quad (5)$$

where ω_{max} is the upper bound of the soft-gluon energy and m is the top quark mass. The factor S_I gives the correction for the vector (the vector and the axial vector) coupling of the top quark to the photon (Z boson). While the factor S_{II} gives the correction of the magnetic moment of the top quark, $\sigma^{\mu\nu} q_\nu$ in the vertices. (The momentum q is the incoming momentum of the photon and the Z boson.) The cross-sections for $e_R^- e_L^+$ are given from Eqns.(1) - (3) by interchanging L, R as well as \uparrow, \downarrow .

Now we come to the discussion of the spin basis which makes the spin correlation maximize. It has been known that there exists the ‘‘Off-Diagonal’’ basis which makes the contributions from the like spin configuration vanish for the $e_L^- e^+ \rightarrow t\bar{t}$ process in the leading order analysis⁵. At the order $\mathcal{O}(\alpha_s)$, we can also employ the Off-Diagonal basis,

$$\tan \xi = \frac{A_{LR}}{B_{LR}} = \frac{(f_{LL} + f_{LR})\sqrt{1-\beta^2} \sin \theta}{f_{LL}(\cos \theta + \beta) + f_{LR}(\cos \theta - \beta)}. \quad (6)$$

This relation is the same as that in the leading order analysis⁵. (The relation for $e_R^- e^+$ is given from Eq. (6) by interchange L and R .) The cross-sections are,

$$\frac{d\sigma}{d\cos\theta}(e_L^- e^+ \rightarrow t_\uparrow \bar{t}_\uparrow \text{ and } t_\downarrow \bar{t}_\downarrow) = 0, \quad (7)$$

$$\frac{d\sigma}{d\cos\theta}(e_L^- e^+ \rightarrow t_\uparrow \bar{t}_\downarrow \text{ or } t_\downarrow \bar{t}_\uparrow) \\ = \left(\frac{3\pi\alpha^2}{4s\beta}\right) (\sqrt{A_{LR}^2 + B_{LR}^2} \mp D_{LR}) [(\sqrt{A_{LR}^2 + B_{LR}^2} \mp D_{LR})(1 + \hat{\alpha}_s S_I) \\ - 2\left(\frac{\gamma^2 A_{LR}^2 + B_{LR} \bar{B}_{LR}}{\sqrt{A_{LR}^2 + B_{LR}^2}} \mp \bar{D}_{LR}\right) \hat{\alpha}_s S_{II}]. \quad (8)$$

The differential cross-sections in the Off-Diagonal basis are shown in Figure 2. The Up-Up ($t_\uparrow \bar{t}_\uparrow$) and the Down-Down ($t_\downarrow \bar{t}_\downarrow$) components are identically zero.

Therefore the total cross-section consists of the Up-Down ($t_{\uparrow}\bar{t}_{\downarrow}$) and the Down-Up ($t_{\downarrow}\bar{t}_{\uparrow}$) components. But the dominant component is only Up-Down ($t_{\uparrow}\bar{t}_{\downarrow}$), which makes up more than 99% of the total cross-section at $\sqrt{s} = 400$ GeV. Although there exists the structure $\sigma^{\mu\nu}q_{\nu}$ in the QCD one-loop correction, it does not change the behavior of the differential cross-section in the Off-Diagonal basis. The QCD correction makes the differential cross-section large by $\sim 20\%$ compared to the tree level cross-section. Thus the Off-Diagonal basis also gives strong correlation to the differential cross-section in the soft-gluon approximation. This means that the Off-Diagonal basis is a really good one. The hard gluon emission, which we do not discuss, leads to the spin flip effects. Therefore it potentially changes the above results. However the spin flip from the real gluon emission is suppressed by $\alpha_s \times (E_g/M_t)$, and the amplitude for emitting a hard gluon is suppressed by the phase space integral. Here E_g is the energy of emitting gluon. So hard gluon emission may not change the above result. In fact, the detailed analysis⁹ suggests that the spin-flip effects from the hard gluon emission are quite small.

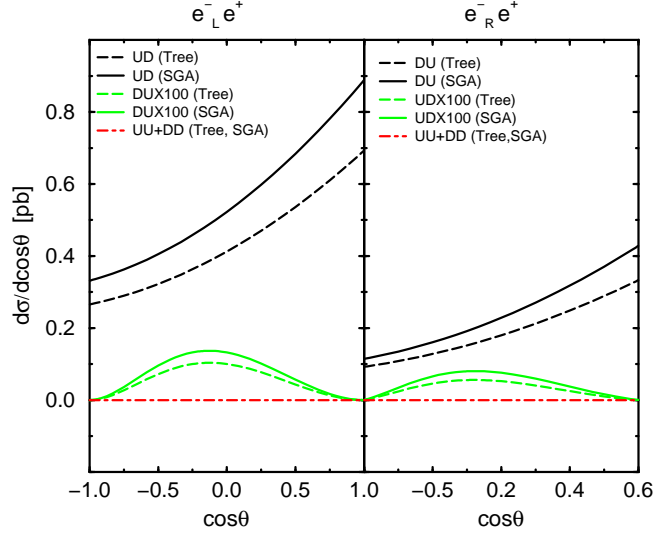


Figure 2: The differential cross-sections in the Off-Diagonal basis at $\sqrt{s} = 400$ GeV, $\omega_{max} = 10$ GeV for the $e^-e^+ \rightarrow t\bar{t}$ process: $t_{\uparrow}\bar{t}_{\downarrow}$ (UD), $t_{\downarrow}\bar{t}_{\uparrow}$ (DU) and $t_{\uparrow}\bar{t}_{\uparrow} + t_{\downarrow}\bar{t}_{\downarrow}$ (UU+DD). The suffix “Tree” and “SGA” mean the differential cross-section at the tree level and at the one-loop level in the soft gluon approximation. We emphasize that DU (UD) component for the $e^-_L e^+$ ($e^-_R e^+$) process is multiplied by 100.

3 Spin Correlations at $\gamma\gamma$ Colliders

At the Next Linear Colliders⁷, we have high energy polarized photon beams which will be produced by the inverse Compton scattering. Therefore the top quark pair production at $\gamma\gamma$ colliders is worthy of investigation. We only discuss the circular polarized photon beams to get clear information on the top and anti-top quark spins, and investigate the cross-section for the process $(\gamma_{R,L}\gamma_{R,L} \rightarrow t_{\uparrow,\downarrow}\bar{t}_{\uparrow,\downarrow})$ in the case of the total angular momentum $J_z = 0, 2$ (for the detail, see Ref.¹⁰). The suffix \uparrow (\downarrow) denotes the spin up (down) for the top and the anti-top quarks and the state γ_R (γ_L) refers to the right-handed (left-handed) photon. For the generic spin basis, we take the same definition as the previous one [Figure 1].

Firstly, we investigate the optimal decomposition of the top pair spins for the $J_z = 0$ channel. The differential cross-sections for this channel are

$$\frac{d\sigma}{d\cos\theta}(\gamma_R \gamma_R \rightarrow t_{\uparrow}\bar{t}_{\uparrow} \text{ or } t_{\downarrow}\bar{t}_{\downarrow}) = y(\beta, \theta) (1 - \beta^2) (1 \mp \beta \cos \xi)^2, \quad (9)$$

$$\frac{d\sigma}{d\cos\theta}(\gamma_R \gamma_R \rightarrow t_{\uparrow}\bar{t}_{\downarrow} \text{ and } t_{\downarrow}\bar{t}_{\uparrow}) = y(\beta, \theta) (1 - \beta^2) \beta^2 \sin^2 \xi. \quad (10)$$

The function $y(\beta, \theta)$ is a common factor, which takes the following form;

$$y(\beta, \theta) \equiv \frac{\beta}{32\pi s} \frac{4N_c(4\pi\alpha)^2 Q_\gamma(t)^4}{(1 - \beta^2 \cos^2 \theta)^2}.$$

Although we can not take the “Off-Diagonal” basis in this case, we can employ the “Diagonal” basis to correlate the top spins strongly, This basis is the familiar Helicity basis. In this basis, we obtain the differential cross-section:

$$\frac{d\sigma}{d\cos\theta}(\gamma_R \gamma_R \rightarrow t_R\bar{t}_R \text{ or } t_L\bar{t}_L) = y(\beta, \theta) (1 - \beta^2) (1 \pm \beta)^2, \quad (11)$$

$$\frac{d\sigma}{d\cos\theta}(\gamma_R \gamma_R \rightarrow t_R\bar{t}_L \text{ and } t_L\bar{t}_R) = 0, \quad (12)$$

where and in what follows $t_{R/L}$ ($\bar{t}_{R/L}$) refers to the top (anti-top) with spin up/down in the “Helicity” basis ($\cos \xi = -1$).

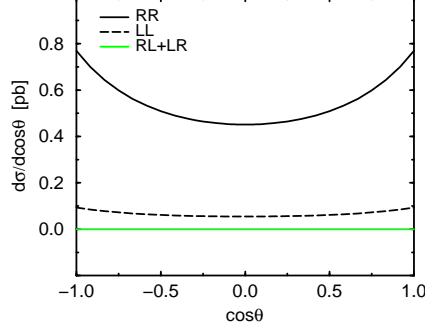


Figure 3: The differential cross-sections for the process $\gamma_R \gamma_R \rightarrow t \bar{t}$ at $\sqrt{s} = 400$ GeV. Each line corresponds to the spin configuration $t_R \bar{t}_R$, $t_L \bar{t}_L$ and $t_R \bar{t}_L + t_L \bar{t}_R$.

In Figure 3 we show the differential cross-section for the $J_z = 0$ channel. The component RR ($t_R \bar{t}_R$) gives the dominate contribution to the total cross-section. The component LL ($t_L \bar{t}_L$) is strongly suppressed by the factor $(1 - \beta)^2$ compared to the $t_R \bar{t}_R$ at $\sqrt{s} = 400$ GeV ($\beta \simeq 0.48$). Roughly speaking, the ratio is $d\sigma(t_R \bar{t}_R) : d\sigma(t_L \bar{t}_L) = 8 : 1$.

Secondly, we discuss the spin correlation for the process $\gamma_R \gamma_L \rightarrow t \bar{t}$ in which the initial angular momentum is $J_z = 2$. The differential cross-sections in generic spin basis are

$$\begin{aligned} & \frac{d\sigma}{d\cos\theta}(\gamma_R \gamma_L \rightarrow t_{\uparrow} \bar{t}_{\uparrow} \text{ and } t_{\downarrow} \bar{t}_{\downarrow}) \\ &= y(\beta, \theta) \beta^2 \sin^2 \theta \left(\sqrt{1 - \beta^2} \sin \theta \cos \xi - \cos \theta \sin \xi \right)^2, \end{aligned} \quad (13)$$

$$\begin{aligned} & \frac{d\sigma}{d\cos\theta}(\gamma_R \gamma_L \rightarrow t_{\uparrow} \bar{t}_{\downarrow} \text{ or } t_{\downarrow} \bar{t}_{\uparrow}) \\ &= y(\beta, \theta) \beta^2 \sin^2 \theta \left(\sqrt{1 - \beta^2} \sin \theta \sin \xi + \cos \theta \cos \xi \mp 1 \right)^2. \end{aligned} \quad (14)$$

In the ‘‘Helicity’’ basis with $\cos \xi = -1$, the differential cross-sections are

$$\frac{d\sigma}{d\cos\theta}(\gamma_R \gamma_L \rightarrow t_{\uparrow} \bar{t}_{\uparrow} \text{ and } t_{\downarrow} \bar{t}_{\downarrow}) = y(\beta, \theta) \beta^2 \sin^4 \theta (1 - \beta^2), \quad (15)$$

$$\frac{d\sigma}{d\cos\theta}(\gamma_R \gamma_L \rightarrow t_{\uparrow} \bar{t}_{\downarrow} \text{ or } t_{\downarrow} \bar{t}_{\uparrow}) = y(\beta, \theta) \beta^2 \sin^2 \theta (\cos \theta \pm 1)^2. \quad (16)$$

Here we can take the ‘‘Off-Diagonal’’ basis by defining the spin angle ξ as follows,

$$\tan \xi = \sqrt{1 - \beta^2} \tan \theta. \quad (17)$$

In this basis, we get the differential cross-sections,

$$\frac{d\sigma}{d\cos\theta}(\gamma_R \gamma_L \rightarrow t_{\uparrow}\bar{t}_{\uparrow} \text{ and } t_{\downarrow}\bar{t}_{\downarrow}) = 0, \quad (18)$$

$$\begin{aligned} \frac{d\sigma}{d\cos\theta}(\gamma_R \gamma_L \rightarrow t_{\uparrow}\bar{t}_{\downarrow} \text{ or } t_{\downarrow}\bar{t}_{\uparrow}) &= y(\beta, \theta) \beta^2 \sin^2 \theta \\ &\times \left(1 \mp \sqrt{1 - \beta^2 \sin^2 \theta}\right)^2. \end{aligned} \quad (19)$$

To see the difference between the Helicity and the Off-Diagonal basis, we plot the differential cross-section at $\sqrt{s} = 400$ GeV [Figure 4]. In the Helicity basis, the cross-section is not dominated by only one spin configuration. While the spin configuration Down-Up ($t_{\downarrow}\bar{t}_{\uparrow}$) dominates the cross-section in the Off-Diagonal basis. Therefore we can uniquely determine the spin configuration of the top quark pairs to be “Down-Up” ($t_{\downarrow}\bar{t}_{\uparrow}$) in the Off-Diagonal basis.

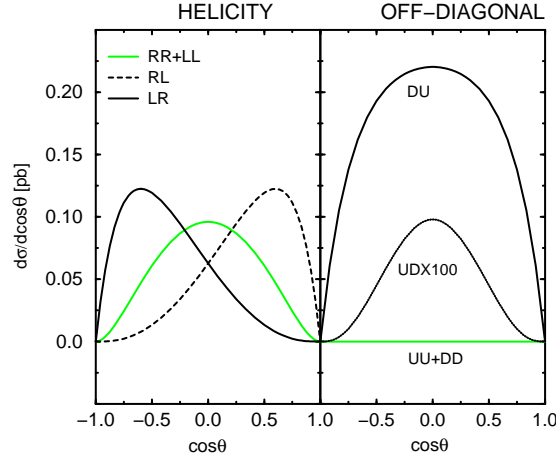


Figure 4: The differential cross-section for the $J_z = 2$ channel ($\gamma_R \gamma_L$) in the Helicity and in the Off-Diagonal bases :RR+LL ($t_R \bar{t}_R + t_L \bar{t}_L$), RL ($t_R \bar{t}_L$) and LR ($t_L \bar{t}_R$) in the Helicity basis. UU+DD ($t_{\uparrow}\bar{t}_{\uparrow} + t_{\downarrow}\bar{t}_{\downarrow}$), UD ($t_{\uparrow}\bar{t}_{\downarrow}$) and DU ($t_{\downarrow}\bar{t}_{\uparrow}$). The UD configuration is multiplied by 100.

4 Summary

Firstly, the top quark pair production at e^+e^- Colliders was discussed. We have investigated the spin correlation at the one-loop level in the soft-gluon

approximation and shown the polarized cross section in the Off-Diagonal basis. In the Off-Diagonal basis, the QCD correction in the soft-gluon approximation does not change the behavior of the cross-section. It only enhances the magnitude of cross-section by $\sim 20\%$ compared to the ones in the leading order analysis. It has been shown that the Off-Diagonal basis is effective even after taking into account the QCD corrections.

Secondly we have discussed the spin correlations at $\gamma\gamma$ Colliders at the tree level in the perturbation theory. The characteristic spin structure which makes spin correlations strong was discussed. In the $J_z = 0$ channel, the Helicity basis is a good basis. The dominant component of the signal is RR ($t_R \bar{t}_R$), which makes up about 90% of the total cross-section at $\sqrt{s} = 400$ GeV. In the $J_z = 2$ channel, we presented the polarized cross-section in the Helicity and Off-Diagonal basis. In the Off-Diagonal basis, only one spin configuration is appreciably different from zero for all values of the scattering angle in contrast with the Helicity basis.

To observe spin correlations at future e^+e^- and the $\gamma\gamma$ Colliders is interesting and may be a good test for the top quark sector of the Standard Model. Especially, the analysis using the Off-Diagonal basis will give us a lot of useful and efficient information.

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